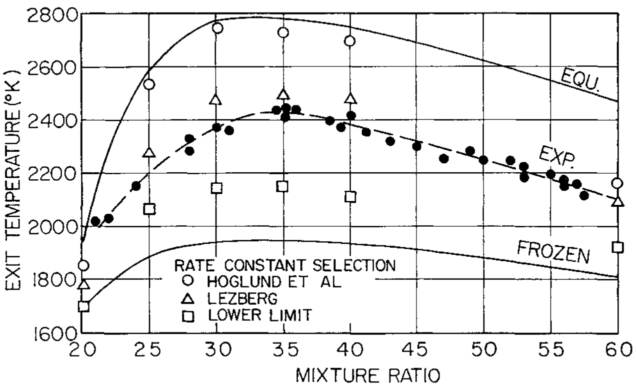

Fig 1 Comparison of experimental and theoretical exit temperatures (RP-1/GO$_2$, ϵ = 5 23:1, P_E = 1 atm)

Since the motor is water-cooled, the exhaust temperature is presumably affected by heat losses from the combustion chamber and nozzle The heat losses for this particular engine were not measured However, losses from a geometrically identical engine burning hydrogen and oxygen as propellants were measured by monitoring cooling water temperature and were found to agree within 20% with those calculated by the method of Bartz [11] Calculations of exit temperatures for the H_2/O_2 engine with the measured heat losses,[7] assuming that all heat is lost from the combustion chamber, lead to a decrease of about 50°K in exit temperature near the stoichiometric mixture ratio for the same operating conditions and freezing pressure

A quick estimate of the heat losses for a stoichiometric RP-1/O_2 system indicates that they are no greater than those for the H_2/O_2 system Since heat losses from regions of the nozzle downstream of the chamber produce a larger effect on exit temperature than equal losses from the chamber, it is expected that the temperatures that were measured on this RP-1/O_2 engine were as much as 100° to 150°K lower than the temperatures that we would have measured if no heat transfer had occurred

The results shown in Fig 1 indicate that, if the reaction scheme proposed by Hoglund et al is valid, then the rate constants suggested by Franciscus and Lezberg give much better agreement with the measured exit temperatures than do either those of Hoglund et al or the "lower limit" values The important reactions and rate constants are

$$H + H + M \rightleftarrows H_2 + M \qquad k_1 = 3 \times 10^{18}\, T^{-1} \qquad (1)$$

$$H + OH + M \rightleftarrows H_2O + M \qquad k_2 = 7\,5 \times 10^{19}\, T^{-1} \qquad (2)$$

$$H + O + M \rightleftarrows OH + M \qquad k_3 \approx 10^{15} - 10^{16} \qquad (3)$$

where k's are in cm^6/mole^2-sec Since k_2 is thought to be considerably larger than k_1 or k_3, and since much more OH than either H or O is present near the stoichiometric mixture ratio, the results of the present investigation should be regarded as an approximate determination of the magnitude of k_2 and as confirmation of the hypothesis that $k_2 \geq 10k_1$ Since a "sudden-freezing" analysis was used in this work and in that of Franciscus and Lezberg, the value of k_2 determined here should be employed only in "sudden-freezing" analyses and not in more detailed calculations [1] Better determination of k_1 could be obtained by running the motor, with hydrogen as fuel, extremely rich, or by an entirely different technique (Many determinations of k_1 by shock-tube techniques have appeared in the literature) A test of the proposed mechanism, which neglects HO_2 as a participant, could be made by running the motor very lean Some runs of this sort have been made but have not been analyzed

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Transpiration and Film Cooling Combined with External Cooling

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Nomenclature

b	=	duct circumference
c_p	=	specific heat
d	=	wall thickness
h	=	convective heat-transfer coefficient
k	=	thermal conductivity
l	=	cooled duct length
q	=	heat flux
w	=	injectant flow rate per unit length of duct circumference
C	=	"effectiveness" of convective coolant
R	=	effectiveness of transpiration coolant
Re	=	Reynolds number of film injection based on slot width
S	=	slot width
T	=	temperature
V	=	velocity
W	=	total injection rate
η	=	effectiveness of film coolant
λ	=	ratio of required injection rates
μ	=	dynamic viscosity
ρ	=	density

Subscripts

a	=	allowable wall temperature
ad	=	adiabatic
c	=	coolant
g	=	gas side
w	=	wall

Superscript

0	=	heat-transfer data with no injection

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IN a recent note Sellers[1] has presented a parametric analysis of the effectiveness of film cooling employed in combination with regenerative cooling of rocket nozzles with hydrogen as a propellant This note compares the effectiveness of two methods of injection cooling, i e , transpiration and film cooling, which may be required to supplement convective cooling of the nozzle in order to prevent the occurrence of wall temperatures in excess of specified permissible temperature levels In particular, we determine the ratio of the injectant flow rates $W_{t\ ansp}$ and W_{film} required to lower the gas-side wall temperature to the allowed level T_a from the level $T_{w\ g}{}^0(\geq T_a)$, which is established by regenerative cooling

The analysis is based on a simplified model of combined injection and regenerative cooling of a duct with constant cross section An elementary expression is obtained for the ratio $W_{t\ ansp}/W_{film}$ in terms of three parameters representing the convection cooling effectiveness C, reduced permissible temperature R_a, and the Reynolds number of film injection Re The simplified analytic results are useful in assessing magnitudes and trends of the coolant injection rate requirements for supplementing the regenerative cooling of rocket nozzles

"Effectiveness" of Convective Cooling

We consider a regeneratively cooled duct of constant cross section with steady-state heat flux through the wall q^0 given by the usual expression for external convection cooling:

$$q^0 = h_g{}^0(T_{g\ ad} - T_{w\ g}{}^0) = \frac{h_g{}^0(T_{g,ad} - T_c{}^0)}{[1 + (h_g{}^0 d_w/k_w) + (h_g{}^0/h\ {}^0)]} \quad (1)$$

It is convenient to define a parameter C representing the effectiveness of the convective coolant:

$$C = (T_{w\ g}{}^0 - T_c{}^0)/(T_{g\ \ d} - T\ {}^0) \quad (2)$$

In view of Eq (1) the effectiveness C may be alternatively expressed in terms of the heat-transfer parameters as

$$C = \frac{[h_g{}^0(d_w/k_w) + (h_g{}^0/h_c{}^0)]}{[1 + h_g{}^0(d_w/k_w) + (h_g{}^0/h_c{}^0)]} \quad (3)$$

For simplicity we shall assume that $h_g{}^0$ is constant corresponding to fully developed turbulent flow in the duct It is assumed further that the thermal resistances d_w/k_w and $h\ {}^0$ are constant, so that C is constant in the duct under consideration Thus, in the subsequent analysis, it will suffice to regard $T_{w,g}{}^0$, $T\ {}^0$, and $T_{g\ ad}$ as preassigned data of the convection cooling contained in the effectiveness parameter C

Transpiration Cooling

The effect of transpiration on the gas-side wall temperature is represented by the empirical correlation for the effectiveness number R in transpiration cooling[2]:

$$R \equiv \frac{T_{w,g} - T_c}{T_{g\ ad} - T} = \left[1 + \frac{(\rho v)_c c_{p,c}}{3h_g{}^0}\right]^{-3} \quad (4)$$

where (ρv) is the injection mass flux and $h_g{}^0$ is the gas-side heat-transfer coefficient that obtains in the absence of injection The correlation equation (4) has been determined by Bartle and Leadon[2] on the basis of extensive experimental investigation

It is to be noted that the effectiveness R is independent of any external cooling parameters other than the temperature T_c of the injectant, which may correspond to that of the regenerative coolant used in transpiration Thus, if $T_{w\ g} = T_a$, the allowed gas-side wall temperature, the required effectiveness

$$R = R_a = (T_a - T\ )/(T_{g\ ad} - T\ ) \quad (5)$$

leads to the following transpiration mass flux requirement:

$$(\rho v) = \frac{3h_g{}^0}{c_p}\left(\frac{1}{R_a{}^{1/3}} - 1\right) \quad (6)$$

Accordingly, the total injection rate for the transpiration cooled wall of area bl is

$$W_{t\ an\ p} = bl\,\frac{3h_g{}^0}{c_p}\left(\frac{1}{R_a{}^{1/3}} - 1\right) \quad (7)$$

independent of external convection cooling

Film Cooling

The effectiveness parameter for film cooling is defined by

$$\eta = (T_{g\ ad} - T_{w\ \ d})/(T_{g\ ad} - T\ ) \quad (8)$$

where $T_{w\ \ d}$ is the adiabatic wall temperature obtained at a station x downstream of the injection port (slot) Various empirical correlations are available for relating η(i e , $T_{w\ ad}$) to the film injection rate w_c, x and properties of the internal gas flow, and of the injectant, as determined in experiments on adiabatic walls The correlations for η show, in general, a monotonic decrease from $\eta = 1$ (i e , $T_{w\ \ d} = T\ $) at the injector to lower values downstream as the film effectiveness decreases toward zero with $T_{w\ \ d} \rightarrow T_{g\ ad}$ In application to externally cooled walls, use is made of $T_{w\ ad}$ by calculating the heat flux from[3]

$$q = h_g{}^0(T_{w\ ad} - T_{w\ g}) \quad (9)$$

where $h_g{}^0$ is the gas-side coefficient that prevails in the absence of injection Thus, if $T_{w\ g} = T_a$ is the permissible gas-side wall temperature for the combined convection and film cooling of the duct wall, we have

$$q = h_g{}^0(T_{w\ ad} - T_a) = \frac{h_g{}^0(T_a - T_c)}{[1 + (h_g{}^0 d_w/k_w) + (h_g{}^0/h_c{}^0)]} \quad (10)$$

In comparing Eq (10) with Eq (1), we may neglect the changes in coolant temperature T due to changes in heat transmission to the coolant and obtain with $T \simeq T_c{}^0$, valid to a high approximation under rocket nozzle heat-transfer conditions,

$$\frac{T_{g,ad} - T_{w,ad}}{T_{g\ ad} - T_c{}^0} = \frac{T_{w,g}{}^0 - T_a}{T_{w\ g}{}^0 - T\ {}^0} = \eta_a \quad (11)$$

The latter result implies that, if $T_{w\ g}{}^0$ is not far in excess of T_a, the supplementary film-cooling requirement can be met with relatively low values of the film-cooling effectiveness $\eta = \eta_a$, corresponding to values of $T_{w,ad}$ close to $T_{g\ \ d}$

The particular form of η employed below is a simplified form of the semiempirical correlation proposed by Hatch and Papell[4] on the basis of extensive experimental data with tangential injection of film coolant:

$$\eta = \exp-\left[\frac{h_g{}^0 x}{w\ c_p} - 0\ 04\right] Re^{\ 1/8} \qquad \text{for } x \geq x_i$$
$$\eta = 1 \qquad \text{for } x \leq x_i \quad (12)$$

where x_i is an initial heat-transfer lag zone defined by

$$h_g{}^0 x_i/w_c c_p = 0\ 04 \quad (13)$$

The simplified representation equation (12) is obtained on the assumption that the coolant injection velocity V is equal to the gas velocity V_g in the duct For $V_g/V_c \neq 1$, an additional factor $f(V_g/V\ ) > 1$, defined in Ref 4, may be introduced in the exponential argument of Eq (12) without any essential complication of the analysis that follows We note further that the representation of the injector effect as Reynolds number, rather than the Peclet number employed in Ref 4, corresponds to the assumption that the coolant Prandtl number is of the order of unity Under typical film-injection conditions, the Reynolds number varies in the range

$$10^2 \lesssim Re = (\rho V/\mu)\,S \lesssim 10^4 \quad (14)$$

where S is the slot width

We now calculate the total film flow rate W_{film} required to maintain $T_{w\,g} \leq T_a$ (i e , $\eta \geq \eta_a$) along a section of the duct of length l (perimeter b) Substituting $\eta = \eta_a$ and $x = l$ into Eq (12), one obtains for injection through a single slot at $x = 0$,

$$W_{film} = bw = bl \frac{h_g}{c_p} \frac{Re^{1/8}}{[0\ 04Re^{1/8} + \ln(1/\eta_a)]} \tag{15}$$

For injection through multiple slots with spacing Δx_j along the flow direction, Eq (12) determines the appropriate $\Delta w_{c\,j}$ required to maintain $\eta \leq \eta_a$ on the assumption that the interference effects in multiple slot injection are negligible Because of the linear relation between Δx_j and $\Delta w_{\ j}$, then, the total injection rate $W = \Sigma \Delta w_{\ j}$ along the length $l = \Sigma \Delta x_j$ is the same as W_{film} in Eq (15) (Actually, $W \lesssim W_{film}$ owing to the beneficial effect of multiple slot interference [5])

Comparison of Total Flow Rates

To effect a comparison of the total flow rate requirements, we note the following relation between η_a and R_a which follows from the use of Eqs (2, 5, and 11) with $T = T^0$:

$$\eta_a = 1 - R_a/C \tag{16}$$

Substituting η_a in Eq (15) for W_{film} and using Eq (7) for W_{transp}, we obtain the ratio

$$\lambda = \frac{W_{transp}}{W_{film}} = 3\left(\frac{1}{R_a^{1/3}} - 1\right) Re_s^{-1/8} \times \left\{0\ 04\, Re^{1/8} + \ln \frac{C}{C - R_a}\right\} \tag{17}$$

The dependence of λ on T_a implicit in R_a is graphically shown in Figs 1 and 2 for injection Reynolds number $Re = 10^2$ and $Re = 10^4$, respectively, with C as a parameter It is seen from these curves that under certain conditions of convective cooling, $C < 1$, the total flow rates required in film cooling are smaller than those required in transpiration cooling, as defined by the criterion $\lambda > 1$ For the adiabatic wall ($C = 1$), λ is always less than unity except in the immediate neighborhood of $R_a \to 0(T_a = T)$ The latter condition can be approached in transpiration cooling only for very large injection rates The occurrence of the asymptotes at $R_a \to C < 1$ is because of the diminishing film-injection requirements as $T_a \to T_{w\,g}{}^0$ Thus, the criterion $\lambda > 1$, corresponding to the regime in which film injection is advantageous, is generally satisfied when only marginal reduction of the wall temperature is required by coolant injection

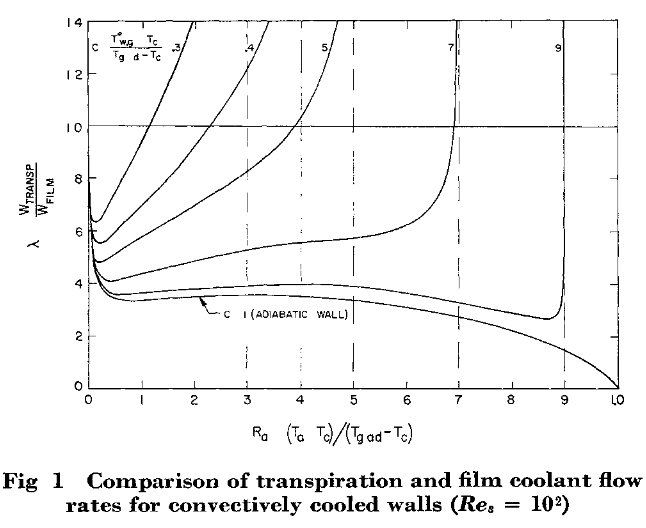

Fig 1 Comparison of transpiration and film coolant flow rates for convectively cooled walls ($Re_s = 10^2$)

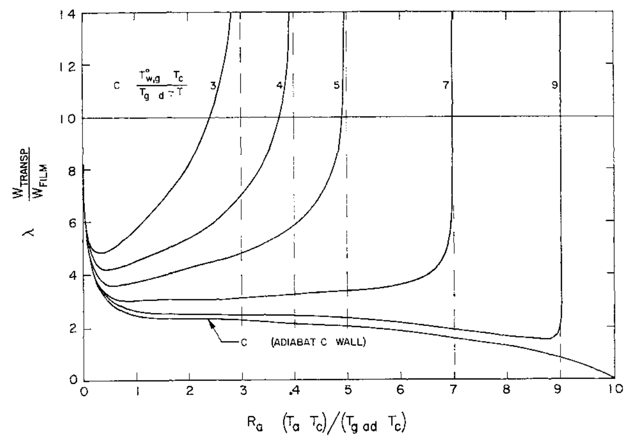

Fig 2 Comparison of transpiration and film coolant flow rates for convectively cooled walls ($Re = 10^4$)

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Maximum Rendezvous Launch Window Characteristics

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Nomenclature

$H\,A$ = hour angle
i = target inclination
i_R = relative inclination
LW = launch window
t_w = wait time
θ_0 = equatorial reference angle
ϕ_L = launch site latitude
ψ = launch azimuth, measured from north
ω = rotational velocity of earth

LAUNCH hold delays, azimuth constraints, and phasing considerations normally preclude the ground launch of a rendezvous resupply vehicle into the plane of a target orbit Ground waiting and/or orbit waiting is therefore a necessary prerequisite to the rendezvous maneuver [1 2]

It is the purpose of this note to define a target orbit inclination that provides the maximum uninterrupted launch window for any particular launch latitude Launch window,

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